

Seesaw Neutrino Signals at the Large Hadron Collider

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Abstract

We discuss the scenario with gauge singlet fermions (right-handed neutrinos) accessible at the energy of the Large Hadron Collider. The singlet fermions generate tiny neutrino masses via the seesaw mechanism and also have sizable couplings to the standard-model particles. We demonstrate that these two facts, which are naively not satisfied simultaneously, are reconciled in the five-dimensional framework in various fashions, which make the seesaw mechanism observable. The collider signal of tri-lepton final states with transverse missing energy is investigated for two explicit examples of the observable seesaw, taking account of three types of neutrino mass spectrum and the constraint from lepton flavor violation. We find by showing the significance of signal discovery that the collider experiment has a potential to find signals of extra dimensions and the origin of small neutrino masses.

I. INTRODUCTION

The neutrino property, in particular its tiny mass scale is one of the most important experimental clues to find new physics beyond the standard model (SM) [1]. From a theoretical viewpoint, the seesaw mechanism [2] has been known to naturally induce small neutrino masses by integrating out new heavy particles (right-handed neutrinos) which interact with the left-handed SM neutrinos. In this Type I seesaw scheme, the right-handed neutrinos have intermediate-scale masses for obtaining $\mathcal{O}(\text{eV})$ light neutrinos. The seesaw effect appears as higher-dimensional operators suppressed by their heavy mass scale and are usually negligible in low-energy effective theory. Alternatively, TeV-scale right-handed neutrinos are found from the seesaw formula to have very weak couplings to the SM sector and their signals would not be naively detected in future experiments such as the CERN Large Hadron Collider (LHC).

It seems therefore difficult to directly observe heavy states which are relevant to suppressing neutrino masses. In the previous work [3], it was pointed out that an observable seesaw mechanism can be implemented in a five-dimensional framework where all the SM fields are confined in a four-dimensional boundary while right-handed neutrinos propagate in the bulk of extra-dimensional space [4, 5]. The existence of extra dimensions is also one of the exciting candidates for new physics beyond the SM [6] and related neutrino phenomenology has been extensively studied in the literature [7]. In the above framework, the right-handed neutrinos and their extra-dimensional partners exist around the TeV scale and have sizable SM gauge and Yukawa couplings in the low-energy effective theory, while the seesaw-induced masses are made small.

The SM neutrinos have tiny masses due to a slight violation of the lepton number. This fact implies that the events with same-sign di-lepton final states [8] may be too rare to be observed unless, e.g., some particular flavor structure is assumed in neutrino mass matrices. In this paper, as in our previous work, we analyze lepton number conserving processes, in particular, the tri-lepton signal with large missing transverse energy: $pp \rightarrow \ell^\pm \ell^\mp \ell^\pm \nu(\bar{\nu})$. This process is expected to be effectively detected at the LHC because only a small fraction of SM processes contributes to the background against the signal. The LHC signatures are studied in typical two types of observable seesaw models in five dimensions and with three types of neutrino mass patterns allowed by the present experimental data [9].

We also present various extra-dimensional approaches which provide the situation that TeV-scale right-handed neutrinos generate a proper scale of seesaw-induced masses and simultaneously have observable interactions to the SM fields. They include boundary Majorana mass terms, boundary conditions for bulk neutrinos, the AdS_5 gravitational background, and their combinations. These scenarios do not rely on particular (singular or aligned) generation structure of mass matrices, and is available even in the one-generation case. For such TeV-scale particles with sizable couplings to the SM sector, the collider experiment will generally have a potential to find a signal of extra dimensions and the origin

of small neutrino masses.

This paper is organized as follows. In Section II, we formulate the general five-dimensional setup for neutrino physics, discussing the seesaw operation and the electroweak Lagrangian. Several explicit models for the observable seesaw are presented for the collider study. In Section III, after the discussion of phenomenological constraints and representative points of model parameters, we numerically investigate the LHC signatures of the seesaw models given in Section II and illustrate the significance for the signal discovery. In Section IV, we further show that various different configurations for the observable seesaw are viable even in one extra dimension, giving the low-energy effective vertices of heavy neutrino fields. Section V is devoted to summarizing our results and discussing future work.

II. GENERAL FRAMEWORK

A. Five-dimensional Setup

Let us consider a five-dimensional theory where the extra space is compactified on the S^1/Z_2 orbifold with the radius R . The SM fields are confined on the four-dimensional boundary at $x^5 = 0$. Besides the gravity, only SM gauge singlets can propagate in the bulk not to violate the charge conservation [4, 5]. The gauge-singlet Dirac fermions Ψ_i ($i = 1, 2, 3$) are introduced in the bulk which contain right-handed neutrinos and their chiral partners. The Lagrangian up to the quadratic order of spinor fields is given by

$$e^{-1}\mathcal{L} = i\bar{\Psi}\not{D}\Psi - \bar{\Psi}(m_d + im_{d5}\gamma_5)\theta(x^5)\Psi - \frac{1}{2}[\bar{\Psi}^c(M + M_5\gamma_5)\Psi + \text{h.c.}]. \quad (2.1)$$

The conjugated spinor is defined as $\Psi^c = i\gamma^2\gamma^0\gamma_5\bar{\Psi}^t$ such that it is Lorentz covariant in five dimensions. The covariant derivative generally contains the contribution of spin connection given by the fünffvein. The bulk Dirac mass term involves the step function $\theta(x^5)$ so that it is invariant under the Z_2 reflection. Such an odd-function dependence could originate from some field expectation value. The bulk mass parameters m_d , m_{d5} , M and M_5 are Z_2 -parity even and generally depend on the extra-dimensional coordinate x^5 which comes from the delta-function dependence (resulting in localized mass terms) and/or the background geometry such as the warp factor in AdS_5 . We also introduce the mass terms between bulk and boundary fields:

$$\mathcal{L}_m = -(\bar{\Psi}mL + \bar{\Psi}^cm^cL)\delta(x^5) + \text{h.c.}, \quad (2.2)$$

where m and m^c denote the mass parameters after the electroweak symmetry breaking (the original Yukawa term will be given later). Throughout this paper, we take the fundamental scale of five-dimensional theory as the unit of mass dimension-ful parameters. The boundary spinors L_i ($i = 1, 2, 3$) contain the left-handed neutrinos ν_i . The Z_2 parity implies that either

component in a Dirac fermion Ψ vanishes at the boundary ($x^5 = 0$) and therefore either of m and m^c becomes irrelevant.¹ In the following we assign the even Z_2 parity to the upper (right-handed) component of bulk fermions

$$\Psi(-x^5) = \gamma_5 \Psi(x^5), \quad (2.3)$$

and will drop the m^c term. In the above, while we only consider the boundary terms at $x^5 = 0$, other boundary terms at $x^5 = \pi R$ can also be written down in the same fashion and have physical relevance on curved backgrounds and/or with complicated field configurations.

With a set of boundary conditions, the bulk fermions Ψ_i are expanded by Kaluza-Klein (KK) modes with their kinetic terms being properly normalized

$$\Psi(x, x^5) = \begin{pmatrix} \sum_n \chi_R^n(x^5) N_R^n(x) \\ \sum_n \chi_L^n(x^5) N_L^n(x) \end{pmatrix}. \quad (2.4)$$

The wavefunctions $\chi_{R,L}^n$ are generally matrix-valued in the generation space and we have omitted the generation indices for notational simplicity. After integrating over the fifth dimension, we obtain the neutrino mass matrix in four-dimensional effective theory:

$$\mathcal{L}_4 = i\mathcal{N}^\dagger \sigma^\mu \partial_\mu \mathcal{N} - \frac{1}{2} (\mathcal{N}^T \epsilon \mathcal{M} \mathcal{N} + \text{h.c.}), \quad (2.5)$$

where $\epsilon = i\sigma^2$, and \mathcal{N} is composed of the boundary neutrinos and the KK modes $\mathcal{N} = (\nu | \epsilon N_R^{0*}, \epsilon N_R^{1*}, N_L^1, \epsilon N_R^{2*}, N_L^2, \dots) \equiv (\nu | N)$. The zero modes of the left-handed components have been extracted according to the boundary condition. The neutrino mass matrix for $(\nu | N)$ is given by

$$\mathcal{M} = \left(\begin{array}{c|cccc} & m_0^t & m_1^t & 0 & \dots \\ \hline m_0 & -M_{R00}^* & -M_{R01}^* & M_{K01} & \dots \\ m_1 & -M_{R10}^* & -M_{R11}^* & M_{K11} & \dots \\ 0 & M_{K10}^t & M_{K11}^t & M_{L11} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array} \right) \equiv \left(\begin{array}{c|c} & M_D^t \\ \hline M_D & M_N \end{array} \right), \quad (2.6)$$

where the Majorana masses $M_{L,R}$, the KK masses M_K , and the boundary Dirac masses m_n are

$$\begin{aligned} M_{R_{mn}} &= \int_{-\pi R}^{\pi R} dx^5 (\chi_R^m)^t (M + M_5) \chi_R^n, & M_{K_{mn}} &= \int_{-\pi R}^{\pi R} dx^5 (\chi_R^m)^\dagger (-\omega \partial_5 + m_d + i m_{d5}) \chi_L^n, \\ M_{L_{mn}} &= \int_{-\pi R}^{\pi R} dx^5 (\chi_L^m)^t (M - M_5) \chi_L^n, & m_n &= \chi_R^{n\dagger}(0) m. \end{aligned} \quad (2.7)$$

¹ The exception is the generation-dependent parity assignment on bulk fields [10]. We do not consider such a possibility in this paper.

In the expression of KK masses M_K , the factor ω is related to the five-dimensional geometry, for example, $\omega = 1$ for the flat background and $\omega = e^{-k|x^5|}$ for the AdS_5 background with the curvature k . It is noticed that $M_{K_{mn}}$ becomes proportional to δ_{mn} if $\chi_{R,L}^n$ are the eigenfunctions of the bulk equations of motion, and $M_{R,L_{mn}}$ also becomes proportional to δ_{mn} if the bulk mass parameters M, M_5 are independent of the coordinate x^5 .

B. Seesaw and Electroweak Lagrangian

We further implement the seesaw operation assuming $\mathcal{O}(m_n) \ll \mathcal{O}(M_K), \mathcal{O}(M_{R,L})$ and find the induced Majorana mass matrix for three-generations light neutrinos²

$$M_\nu = -M_D^t M_N^{-1} M_D. \quad (2.8)$$

It is useful for later discussion of collider phenomenology to write down the electroweak Lagrangian in the basis where all the mass matrices are generation diagonalized. The original Lagrangian of four-dimensional neutrinos comes from (2.5) and the SM part. The kinetic and mass terms and the interactions to the electroweak gauge bosons are given in the mass eigenstate basis (ν_d, N_d) as follows:

$$\begin{aligned} \mathcal{L}_{\text{EW}} = & i\nu_d^\dagger \sigma^\mu \partial_\mu \nu_d + iN_d^\dagger \sigma^\mu \partial_\mu N_d - \frac{1}{2}(\nu_d^\dagger \epsilon M_\nu^d \nu_d + N_d^\dagger \epsilon M_N^d N_d + \text{h.c.}) \\ & + \frac{g}{\sqrt{2}} \left[W_\mu^\dagger e^\dagger \sigma^\mu U_{\text{MNS}} (\nu_d + V N_d) + \text{h.c.} \right] \\ & + \frac{g}{2 \cos \theta_W} Z_\mu (\nu_d^\dagger + N_d^\dagger V^\dagger) \sigma^\mu (\nu_d + V N_d), \end{aligned}$$

where W_μ and Z_μ are the electroweak gauge bosons and g is the $SU(2)_{\text{weak}}$ gauge coupling constant. The 2-component spinors ν_d are three light neutrinos for which the seesaw-induced mass matrix M_ν is diagonalized

$$M_\nu^d = U_\nu^t M_\nu U_\nu, \quad U_\nu \nu_d = \nu - M_D^\dagger M_N^{-1*} N, \quad (2.9)$$

and N_d denote the infinite number of neutrino KK modes for which the bulk Majorana mass matrix M_N is diagonalized both in the generation and KK spaces by a unitary matrix U_N :

$$M_N^d = U_N^t M_N^d U_N, \quad U_N N_d = N + M_N^{-1} M_D \nu. \quad (2.10)$$

The lepton mixing matrix measured in the neutrino oscillation experiments is given by $U_{\text{MNS}} = U_e^\dagger U_\nu$ where U_e is the left-handed rotation matrix for diagonalizing the charged-lepton Dirac masses. It is interesting to find that the model-dependent parts of electroweak

² In theory with more than one extra dimensions, this matrix product (the sum of infinite KK modes) generally diverge without some regularization [12].

gauge vertices are governed by a single matrix V which is defined as

$$V = U_\nu^\dagger M_D^\dagger M_N^{-1*} U_N. \quad (2.11)$$

When one works in the basis where the charged-lepton sector is flavor diagonalized, U_ν is fixed by the neutrino oscillation matrix.

The neutrinos also have the interaction to the electroweak doublet Higgs H . If assuming H lives in the four-dimensional fixed point at $x^5 = 0$, the boundary Dirac mass (2.2) comes from the Yukawa coupling

$$\mathcal{L}_h = -y \tilde{H}^\dagger \bar{\Psi} L \delta(x^5) + \text{h.c.}, \quad (2.12)$$

where $\tilde{H} = \epsilon H^*$. The doublet Higgs H has a non-vanishing expectation value $v/\sqrt{2}$ and its fluctuation $h(x)$. After integrating out the fifth dimension and diagonalizing mass matrices, we have

$$\mathcal{L}_h = \frac{-h}{v} \sum_n [(N_d^t - \nu_d^t V^*) U_N^t]_{R_n} m_n U_\nu \epsilon(\nu_d + V N_d) + \text{h.c.}, \quad (2.13)$$

where $[\cdots]_{R_n}$ means the n -th mode of the right-handed component.

C. Models for Observable Seesaw

The heavy neutrino interactions to the SM fields are determined by the mixing matrix V both in the gauge and Higgs vertices. The $3 \times \infty$ matrix V is determined by the matrix forms of neutrino masses in the original Lagrangian $\mathcal{L} + \mathcal{L}_m$. The matrix elements in V have the experimental upper bounds from electroweak physics, as will be seen later. Another important constraint on V comes from the low-energy neutrino experiments, namely, the seesaw-induced masses should be of the order of eV scale, which in turn specifies the scale of heavy neutrino masses M_N . This can be seen from the definition of V by rewriting it with the light and heavy neutrino mass eigenvalues

$$V = i(M_\nu^d)^{\frac{1}{2}} P (M_N^d)^{-\frac{1}{2}}, \quad (2.14)$$

where P is an arbitrary $3 \times \infty$ matrix with $P P^t = 1$. Therefore one naively expects that, with a fixed order of $M_\nu^d \sim 10^{-1}$ eV and $|V| \gtrsim 10^{-2}$ for the discovery of experimental signatures of heavy neutrinos, their masses should be very light and satisfy $M_N^d \lesssim \text{keV}$ (this does not necessarily mean the seesaw operation is not justified as M_ν^d is fixed). The previous collider studies on TeV-scale right-handed neutrinos [11] did not impose the seesaw relation (2.14) and have to rely on some assumptions for suppressing the necessarily induced masses M_ν . For example, the neutrino mass matrix has some singular generation structure, otherwise it leads to the decoupling of seesaw neutrinos from collider physics.

We here present two scenarios in which heavy neutrino modes are accessible at future colliders. The numerical study of these two models will be performed in the next section. It is noted that they are illustrative examples and there are many other possibilities for the observable seesaw with extra dimensions. We will comment on such various alternatives in a later section.

1. Model 1 – Particular Majorana Masses –

A possible scenario for observable heavy neutrinos is to take a specific value of Majorana mass parameters. Let us consider the situation that the bulk Majorana mass M and bulk Dirac masses are vanishing on the Minkowski background. The Lagrangian is

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi - \left[\frac{1}{2}\bar{\Psi}^c M_5 \gamma_5 \Psi + \bar{\Psi} m L \delta(x^5) + \text{h.c.} \right]. \quad (2.15)$$

The equations of motion without bulk masses are solved by simple oscillators and the mass matrices in four-dimensional effective theory (2.5) are found

$$M_{Kmn} = -\frac{n}{R}\delta_{mn}, \quad M_{Rmn} = -M_{Lmn} = M_5\delta_{mn}, \quad m_n = \frac{m}{\sqrt{2\delta_{n0}\pi R}}. \quad (2.16)$$

From these, we find the seesaw-induced mass matrix and the mixing with heavy modes:

$$M_\nu = \frac{1}{2\pi R} m^\dagger \frac{\pi R |M_5|}{\tan(\pi R |M_5|)} \frac{1}{M_5^*} m, \quad (2.17)$$

$$\nu = U_\nu \nu_d - \frac{m^\dagger}{\sqrt{2\pi R}} \left[\frac{1}{M_5} \epsilon N_R^{0*} + \sum_{n=1} \frac{\sqrt{2}}{|M_5|^2 - (n/R)^2} \left(M_5^* \epsilon N_R^{n*} - \frac{n}{R} N_L^n \right) \right]. \quad (2.18)$$

The KK neutrinos have the mass eigenvalues $|M_5|$ and $\frac{n}{R} \pm |M_5|$ ($n \geq 1$). The effect of infinitely many numbers of KK neutrinos appears as the additional factor $\pi R |M_5| / \tan(\pi R |M_5|)$. An interesting case is that (the eigenvalues of) M_5 takes a specific value $|M_5| = \alpha/R$ where α contains half integers [4]: the seesaw-induced mass M_ν is then suppressed by the tangent factor (not only by large Majorana mass), on the other hand, the heavy mode vertex V is un-suppressed. This fact realizes the situation that right-handed neutrinos in the seesaw mechanism are observable at sizable rates in future collider experiments [3] (see also [13]).

As an explicit example, we consider flavor-independent Majorana masses $M_5 = \frac{1}{2R} - \delta_M$ where δ_M is small ($\ll 1/R$) and denotes a deviation from massless neutrinos. A vanishing δ_M makes the light neutrinos exactly massless, where a complete cancellation occurs within the seesaw effects of heavy neutrinos. As we will see, the parameter δ_M takes a tiny value

for giving the correct neutrino mass scale.³ In the KK-mode picture, the mass spectrum becomes almost vector-like and no chiral zero mode exists. The seesaw-induced mass and the KK Dirac masses M_n are given by

$$M_\nu = \frac{\pi R \delta_M}{2} m^\dagger m, \quad M_n \simeq \frac{n - \frac{1}{2}}{R} \quad (n \geq 1). \quad (2.19)$$

We will consider $M_n \sim 1/R = \mathcal{O}(10^{2-3})$ GeV for the LHC analysis of low-lying KK neutrinos. The neutrino Yukawa coupling y_ν is expressed as

$$y_\nu = \frac{2}{\pi R v} (\delta_M)^{\frac{-1}{2}} O^\dagger (M_\nu^d)^{\frac{1}{2}} U_{\text{MNS}}^\dagger, \quad (2.20)$$

where O is an arbitrary 3×3 orthogonal matrix which generally comes in reconstructing high-energy quantities from the low-energy neutrino observables [14]. That corresponds to the matrix P in (2.14).

2. Model 2 – Light Dirac Neutrinos –

Another example of observable heavy states is realized by assuming no Majorana mass for bulk neutrinos, which leads to lepton number conservation while having sizable couplings to the SM neutrinos. The Lagrangian is

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi - \bar{\Psi}m_d\theta(x^5)\Psi - [\bar{\Psi}mL\delta(x^5) + \text{h.c.}]. \quad (2.21)$$

The solution to the bulk equations of motion in the presence of bulk Dirac masses are given by

$$\chi_R^0 = \frac{1}{\sqrt{\pi R}} f_0 e^{-m_d|x^5|}, \quad (2.22)$$

$$\chi_R^n = \frac{1}{\sqrt{\pi R}} \left[f_n \cos\left(\frac{n}{R}x^5\right) + \sqrt{1 - f_n^2} \theta(x^5) \sin\left(\frac{n}{R}x^5\right) \right], \quad (2.23)$$

$$\chi_L^n = \frac{1}{\sqrt{\pi R}} \sin\left(\frac{n}{R}x^5\right), \quad (2.24)$$

$$f_0 = \sqrt{\frac{\pi R m_d}{1 - e^{-2\pi R m_d}}}, \quad f_n = \frac{-n/R}{\sqrt{(n/R)^2 + m_d^2}} \quad (n \geq 1). \quad (2.25)$$

³ That seems a fine tuning of model parameters; the bulk Majorana masses must be fixed almost exactly. This tuning is ameliorated by considering a different extra-dimensional setup with the same neutrino mass matrix (see Section IV B).

The zero mode N_R^0 is massless at this stage and has a localized wavefunction controlled by the bulk Dirac mass m_d . The n -th excited modes have the squared mass eigenvalues $(n/R)^2 + m_d^2$. The mass matrices in four-dimensional effective theory (2.5) are found

$$M_{K_{mn}} = \sqrt{(n/R)^2 + m_d^2} \delta_{mn}, \quad M_{R_{mn}} = M_{L_{mn}} = 0, \quad m_n = \frac{f_n}{\sqrt{\pi R}} m. \quad (2.26)$$

While the excited modes are heavy ($\gtrsim 1/R$), the zero mode has no contribution from bulk and KK masses. Therefore the zero modes compose of Dirac particles with the SM neutrinos and obtain their masses from the SM Higgs field: $\mathcal{L} = -m_0 N_R^0 \nu + \text{h.c.}$. On the other hand, since the excited modes have KK Dirac masses and no lepton number violation, they do not give rise to the seesaw-induced mass M_ν (i.e. the contributions from N_R^n and N_L^n are cancelled to each other) and the right-handed components N_R^n do not mix with the left-handed SM neutrinos. We thus find the light Dirac neutrino masses and the mixing with heavy modes:

$$m_0 = \sqrt{\frac{m_d}{1 - e^{-2\pi R m_d}}} m, \quad (2.27)$$

$$\nu = U_\nu \nu_d - \frac{m^\dagger}{\sqrt{\pi R}} \sum_{n=1} \frac{n/R}{(n/R)^2 + m_d^2} N_L^n. \quad (2.28)$$

The Dirac neutrino mass m_0 can be suppressed by the exponential wavefunction factor f_0 , while the heavy KK modes are kept observable. For example, if $-Rm_d \sim 8$, the $\mathcal{O}(\text{eV})$ neutrinos are obtained for other parameters being on TeV scale. A negative value of m_d means that the zero mode is localized away from the SM boundary ($x^5 = 0$), which situation leads to the suppression of Dirac neutrino mass m_0 . The heavy modes have rather broad wavefunctions in the bulk and the couplings to the SM sector are independent of the exponential suppression. That allows the heavy modes to take sizable boundary couplings and to be observed.

In this model, the light and KK neutrinos are all Dirac particles and their mass eigenvalues are given by

$$M_\nu (= m_0) = \sqrt{\frac{m_d}{1 - e^{-2\pi R m_d}}} m, \quad M_n = \sqrt{(n/R)^2 + m_d^2} \quad (n \geq 1). \quad (2.29)$$

We will consider $M_n \sim m_d = \mathcal{O}(10^{2-3})$ GeV for the LHC analysis of low-lying KK neutrinos. The neutrino Yukawa coupling y_ν is expressed as

$$y_\nu = \frac{2}{v} \sqrt{\frac{1 - e^{-2\pi R m_d}}{2m_d}} U_R M_\nu^d U_{\text{MNS}}^\dagger, \quad (2.30)$$

where U_R is the 3×3 unitary matrix which rotates the three-generation right-handed zero modes so that the light Dirac mass matrix M_ν is diagonalized to M_ν^d .

3. Model 3 – Small Lepton Number Violation –

A slightly different model for observable heavy neutrinos is constructed by introducing small bulk Majorana mass into Model 2, which means the light neutrinos are Majorana particles. The Lagrangian is

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi - \bar{\Psi}m_d\theta(x^5)\Psi - \left[\frac{1}{2}\bar{\Psi}^c M\Psi + \bar{\Psi}mL\delta(x^5) + \text{h.c.}\right]. \quad (2.31)$$

With non-vanishing Majorana masses, the lepton number is broken and the seesaw-induced mass M_ν is generated by the integration of heavy modes as in Model 1;

$$M_\nu = \frac{1}{2\pi R} m^\dagger \left[\pi R m_d + \frac{\pi R \sqrt{|M|^2 + m_d^2}}{\tanh(\pi R \sqrt{|M|^2 + m_d^2})} \right] \frac{1}{M^*} m. \quad (2.32)$$

The mixing with KK neutrinos has a similar expression to Model 2;

$$\nu \simeq U_\nu \nu_d - \frac{m^\dagger}{\sqrt{\pi R}} \left[\frac{f_0}{\sqrt{2}M} \epsilon N_R^{0*} + \sum_{n=1} \frac{n/R}{(n/R)^2 + m_d^2} N_L^n \right], \quad (2.33)$$

where we have assumed $|M| \ll 1/R, |m_d|$. The zero-mode contribution is suppressed if it is enough separated from the SM boundary with the localizing wavefunction, which implies $-Rm_d \gtrsim 6$. It is found from the above expressions that the seesaw neutrino mass and the couplings of heavy modes can be determined independently that makes the seesaw mechanism observable. The neutrino mass, i.e. the size of lepton number violation is controlled by the bulk Majorana mass M . For example, if $-Rm_d \sim 6$ and a few % mixing of heavy mode,

$$M \sim 10^3 \text{ eV}, \quad (2.34)$$

for eV seesaw-induced masses. The fundamental scale of five-dimensional theory is irrelevant for this evaluation. The zero mode N_R^0 obtains a Majorana mass of the order of M and is a light isolated particle with a negligible interaction to the SM sector.

In the end, the low-energy theory contains light Majorana neutrinos with the seesaw-induced mass M_ν , almost decoupled zero modes with mass around keV scale, and heavy KK Dirac neutrinos. The mass eigenvalues of these states are explicitly given by

$$M_\nu \simeq m^\dagger \left(\frac{M}{4m_d} \right) m, \quad M_0 = M, \quad M_n \simeq \sqrt{(n/R)^2 + m_d^2} \quad (n \geq 1). \quad (2.35)$$

The low-lying KK states would be observable at colliders for $M_n \sim m_d = \mathcal{O}(10^{2-3})$ GeV. The neutrino Yukawa coupling has a similar expression to that in Model 1.

III. SEESAW SIGNATURES AT THE LHC

The production of KK-excited neutrino states is the most important signal in our scenarios, since the signal enables us to explore the mechanism responsible for the generation

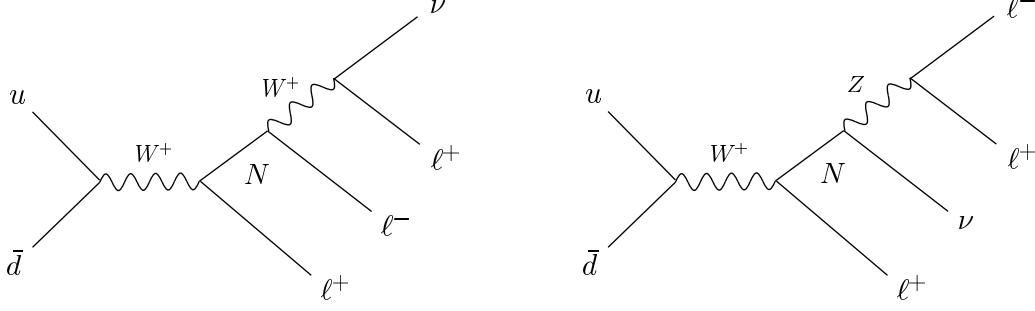


FIG. 1: The lepton number preserving tri-lepton processes at the LHC.

of tiny neutrino masses. An immediate question is which processes we should pay attention to find out the signal at the LHC. As shown in the previous work [3], the tri-lepton signal with missing transverse energy is most prominent since only a small fraction of SM processes contributes to the background against the signal. This lepton number conserving processes, $pp \rightarrow \ell^\pm \ell^\mp \ell^\pm \nu(\bar{\nu})$, dominantly occur through the diagrams shown in FIG. 1. In this section, we investigate such seesaw signatures in Models 1 and 2, which are presented in the previous section as typical examples of the observable seesaw. In the following simulation study, we assume, for simplicity, that the bulk mass parameters of right-handed neutrinos are common in flavor space and the complex phases vanish in the orthogonal matrix O . With these assumptions, we perform the numerical analysis for the tri-lepton signal in various mass hierarchies of neutrino masses, that is, the normal, inverted, and degenerated patterns.

A. Constraints on Neutrino Yukawa Couplings

Before going to discuss the simulation study in details, we summarize the neutrino mass and mixing matrices which are mandatory to investigate the collider signatures at the LHC. The two matrices are parameterized as

$$\begin{aligned}
 M_\nu^d &= \begin{pmatrix} m_{\nu_1} & & \\ & m_{\nu_2} & \\ & & m_{\nu_3} \end{pmatrix}, & \phi &= \begin{pmatrix} e^{i\varphi_1} & & \\ & e^{i\varphi_2} & \\ & & 1 \end{pmatrix}, \\
 U_{\text{MNS}} &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \phi, \quad (3.1)
 \end{aligned}$$

where s_x (c_x) means $\sin \theta_x$ ($\cos \theta_x$). The Dirac and Majorana phases are denoted by δ and $\varphi_{1,2}$, respectively. Note that Majorana phases are not relevant in Model 2 since there is no Majorana mass term in the neutrino sector. The neutrino mass differences and the

	$m_{\nu 1}$	$m_{\nu 2}$	$m_{\nu 3}$
Normal	0	Δm_{21}	$\Delta m_{21} + \Delta m_{32}$
Inverted	$\Delta m_{32} - \Delta m_{21}$	Δm_{32}	0
Degenerate	m_{tot}	$m_{\text{tot}} + \Delta m_{21}$	$m_{\text{tot}} + \Delta m_{21} + \Delta m_{32}$

TABLE I: Three types of neutrino mass hierarchies used in the simulation study.

generation mixing parameters have been measured at neutrino oscillation experiments [9]. We take their typical values,

$$\Delta m_{21} \equiv m_{\nu 2} - m_{\nu 1} = 9 \times 10^{-3} \text{ eV}, \quad (3.2)$$

$$\Delta m_{32} \equiv |m_{\nu 3} - m_{\nu 2}| = 5 \times 10^{-2} \text{ eV}, \quad (3.3)$$

$$s_{12} = 0.56, \quad s_{23} = 0.71, \quad s_{13} \leq 0.22. \quad (3.4)$$

The neutrino mass spectrum is allowed to have three different types of hierarchies and is summarized in TABLE I, where we define $m_{\text{tot}} = (0.67 \text{ eV} - 2\Delta m_{21} - \Delta m_{32})/3 \simeq 0.2 \text{ eV}$, taking account of the cosmological bound: $\sum_i m_{\nu i} \leq 0.67 \text{ eV}$ [15].

Since the scenarios we are studying also affect several physical observables such as the flavor-changing processes of charged leptons [16], it is important to consider the constraints on neutrino Yukawa couplings to have proper representative points. Integrating out all heavy KK neutrinos, we obtain the following dimension 6 operator $\mathcal{O}^{(6)}$ in low-energy effective theory which contributes to the leptonic flavor-changing neutral current;

$$\mathcal{O}^{(6)} = \frac{1}{v^2} (\bar{L} \tilde{H}) \epsilon_N i \not{\partial} (\tilde{H}^\dagger L), \quad (3.5)$$

where the coefficient matrix ϵ_N ($\epsilon_N^{(1)}$ and $\epsilon_N^{(2)}$ for Models 1 and 2) turns out to be

$$\epsilon_N^{(1)} = \frac{2}{\delta_M} U_{\text{MNS}} M_\nu^d U_{\text{MNS}}^\dagger, \quad (3.6)$$

$$\epsilon_N^{(2)} = \frac{e^{-\pi R m_d}}{2m_d^2} \left[\cosh(\pi R m_d) - \frac{\pi R m_d}{\sinh(\pi R m_d)} \right] U_{\text{MNS}} (M_\nu^d)^2 U_{\text{MNS}}^\dagger. \quad (3.7)$$

The operator $\mathcal{O}^{(6)}$ receives phenomenological constraints as shown in Ref. [17], and each component of neutrino Yukawa couplings is thus restricted by comparing the model predictions of the coefficient with experimental data. In particular, the most severe limit is given by the 1-2 component, i.e., the $\mu \rightarrow e\gamma$ search which puts on the upper bound more than 3 orders of magnitude stronger than the others. To weaken the bound on this operator, especially for the 1-2 component, we take representative values of lepton mixing matrix as shown in TABLE II. As a result, new physics parameters in the coefficient ϵ_N such as δ_M

	Model 1			Model 2	
	s_{13}	δ	$\varphi_{1,2}$	s_{13}	δ
Normal	0.07	π	0	0.006	π
Inverted	0.09	0	0	0.19	0
Degenerate	0.04	π	0	0.05	π

TABLE II: The representative points for U_{MNS} in Models 1 and 2. The Majorana phases in U_{MNS} have no physical relevance in the present work and are set to be zero.

in Model 1 and Rm_d in Model 2 turn out to be constrained as follows for each pattern of neutrino mass hierarchy:

$$\begin{aligned}
& \text{(Model 1)} \quad \delta_M \geq 3.3 \text{ eV} \quad \text{for Normal,} \\
& \quad \delta_M \geq 4.4 \text{ eV} \quad \text{for Inverted,} \\
& \quad \delta_M \geq 24. \text{ eV} \quad \text{for Degenerate,} \\
& \text{(Model 2)} \quad -Rm_d \leq 8.5 - 9.0 \quad \text{for Normal,} \\
& \quad -Rm_d \leq 8.5 - 8.9 \quad \text{for Inverted,} \\
& \quad -Rm_d \leq 8.0 - 8.4 \quad \text{for Degenerate.}
\end{aligned} \tag{3.8}$$

Notice that the coefficient ϵ_N in Model 1 is irrelevant to the compactification radius R and so the bounds on δ_M are. For Model 2, the above bounds are obtained for $1/R = 100 - 350 \text{ GeV}$. The compactification radius is also limited by the LEP experiment through the masses of KK excited neutrinos. The lightest ones are $M_1 = 1/(2R)$ for Model 1 and $M_1 = \sqrt{1/R^2 + m_d^2}$ for Model 2, and these states have not be experimentally detected so far. We numerically checked that the constraint is not so severe if $M_1 > 150 \text{ GeV}$. Finally, the SM Higgs mass is to be $m_h = 120 \text{ GeV}$ in evaluating the decay widths of heavy KK neutrinos.

B. Tri-lepton Signals at the LHC

Now let us investigate the tri-lepton signal of heavy neutrino productions at the LHC. Since the tau lepton is hardly detected compared to the others, we consider the signal event including only electrons and muons. There are four kinds of tri-lepton signals: eee , $ee\mu$, $e\mu\mu$, and $\mu\mu\mu$. In this work, we use two combined signals which are composed of $eee + ee\mu$ (the $2e$ signal) and $e\mu\mu + \mu\mu\mu$ (the 2μ signal). Figure 2 shows the total cross sections for these signals from the 1st KK neutrino production at the LHC, which are described as the functions of their mass eigenvalues with fixed values of ϵ_N . It is found from the figure that the cross sections have the universal behaviour within extra-dimensional models; for the normal

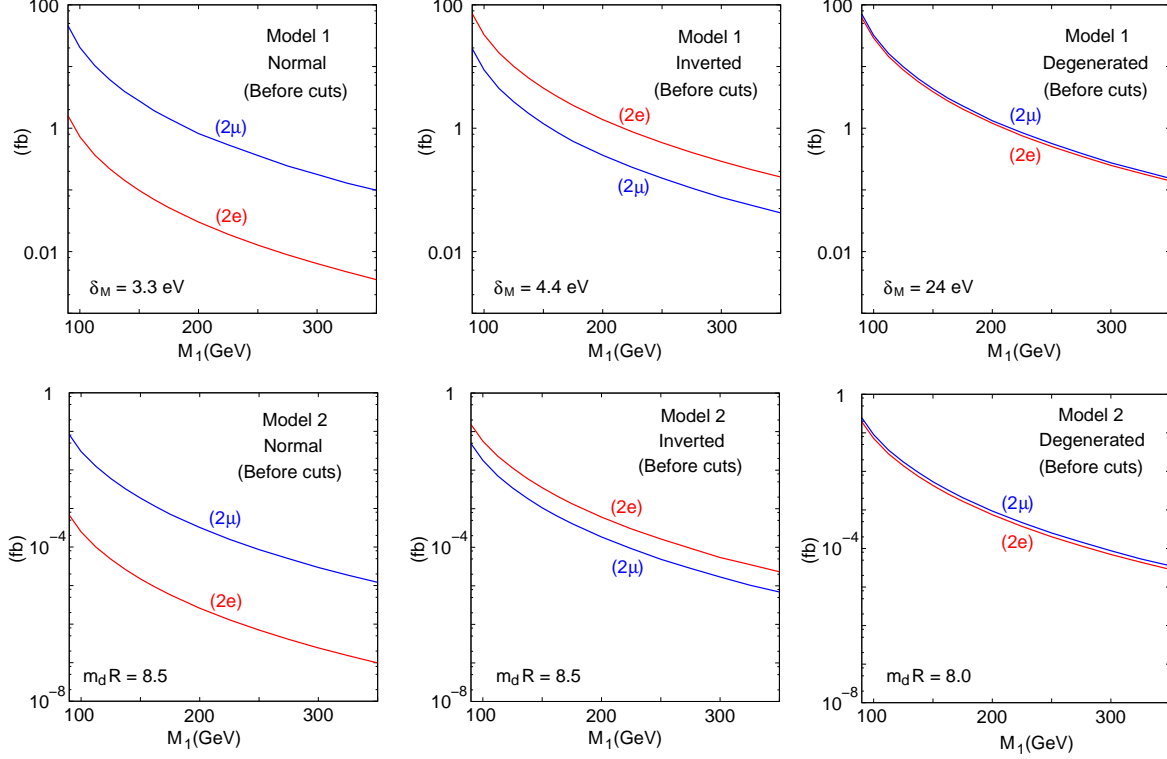


FIG. 2: Total cross sections of tri-lepton signals with fixed values of ϵ_N . The horizontal axes are the masses of the lightest KK-excited modes. The upper and lower panels are for Models 1 and 2, respectively.

mass hierarchy, the cross section for the 2μ signal is about one or two orders of magnitude larger than the $2e$ signal, and for the inverted and degenerate spectra, the $2e$ signal cross section becomes larger than or almost equal to the 2μ one. Further, the cross section for Model 2 is found to be small compared with that for Model 1. This is due to a small wavefunction factor of low-lying KK neutrino mode, $f_1 \sim 1/(Rm_d)$, which is suppressed by $\ln(M_\nu/v)$ to have tiny neutrino masses. We have also evaluated the contributions of tri-lepton signals from heavier KK neutrinos and found that they are small by more than one order of magnitude and are out of reach of the LHC experiment. A high luminosity collider with clean environment such as the International Linear Collider would distinctly discover the signatures of KK mode resonances.

To clarify whether the tri-lepton signal is captured at the LHC, it is important to estimate SM backgrounds against the signal. The SM backgrounds which produce or mimic the tri-lepton final state have been studied [18, 19] and for the present purpose a useful kinematical cut is discussed to reduce these SM processes [19]. According to that work, we adopt the following kinematical cuts for both Models: (i) the existence of two like-sign charged leptons

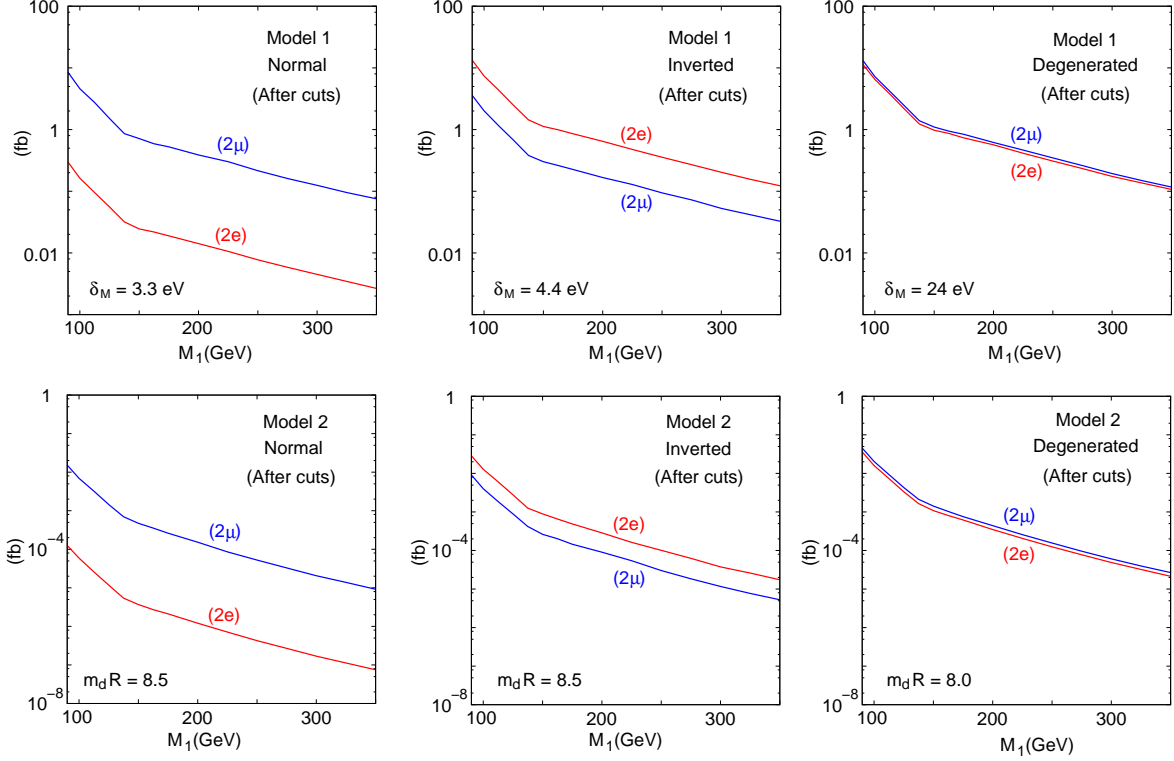


FIG. 3: Total cross sections of tri-lepton signals with fixed values of ϵ_N after implementing the kinematical cuts (see the text). The horizontal axes are the masses of the lightest KK-excited modes. The upper and lower panels are for Models 1 and 2, respectively.

ℓ_1^\pm , ℓ_2^\pm , and an additional one with the opposite charge ℓ_3^\mp , (ii) both of the energies of like-sign leptons are larger than 30 GeV, and (iii) the invariant masses from ℓ_1 and ℓ_3 and from ℓ_2 and ℓ_3 are larger than $m_Z + 10$ GeV or smaller than $m_Z - 10$ GeV. The last condition is imposed to reduce the large background from the leptonic decays of Z bosons in the SM processes. Figure 3 shows the total cross sections of signals after imposing these kinematical cuts. To estimate the efficiency for the signal events due to the cuts, we use the Monte Carlo simulation using the CalcHep code [20]. Since the event numbers of SM backgrounds after the cuts are about 260 for the $2e$ signal and 110 for the 2μ one with the luminosity of 30 fb^{-1} [19], the 2μ events are expected to be observed if the lightest KK mass M_1 is less than a few hundred GeV.

For Model 1, FIG. 4 shows the luminosity which is required to find the seesaw neutrino signal at the LHC as the contour plots on the parameter plane. The luminosity contours for 10, 30, and 300 fb^{-1} are depicted in the figures. These contours are obtained by computing

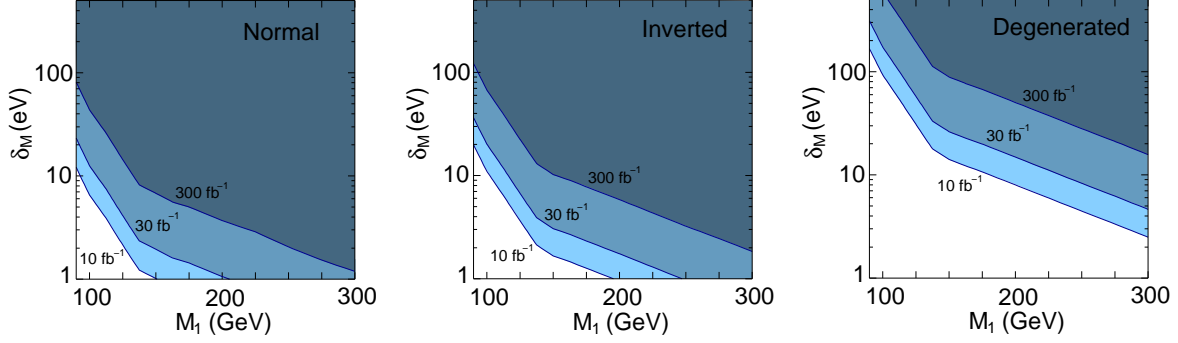


FIG. 4: Luminosity for the 3σ reach in Model 1. The 10, 30, and 300 fb^{-1} contours are shown on the (M_1, δ_M) plane for three types of neutrino mass hierarchy.

the significance for the signal discovery

$$S = \sqrt{S_e^2 + S_\mu^2}, \quad S_i = \frac{N_{S_i}}{\sqrt{N_{S_i} + N_{B_i}}} \quad (i = e, \mu), \quad (3.10)$$

where N_S (N_B) denotes the total number of the $2e$ or 2μ events (that of the corresponding SM backgrounds) after the kinematical cuts. Since both N_S and N_B are proportional to the luminosity, it is possible to estimate the required luminosity for, e.g. giving $S = 3$, which is plotted in the above figures. The luminosity for signal confirmation ($S \geq 5$) are also found by rescaling the results, according to the formula (luminosity) $\propto S^2$. It is found that, if M_1 is less than a few hundreds GeV, the signals would be observed at an early run of the LHC, in particular, Model 1 with the degenerate mass spectrum will definitely be excluded or confirmed. A larger luminosity is needed for a smaller size of extra dimension to reveal its existence.

Model 2, as it stands, generally predicts too small production rates to be found out at the LHC. However the extra-dimensional framework has various options without introducing additional particles. That leads to simple modifications of the model and can make it observable, as explained explicitly in the next section.

IV. OTHER CONFIGURATIONS FOR OBSERVABLE SEESAW

We have discussed the collider signatures of two typical models of observable seesaw. They are constructed in five-dimensional spacetime and utilize the mechanisms which are peculiar to the presence of extra dimensions; the specific value of bulk Majorana mass in Model 1 and the suppression factor from localized wavefunction in Model 2. As we mentioned before, while the Lagrangian is simple and common, the five-dimensional theory makes right-handed

neutrinos observable in various ways which have their own physical meanings. Among them, we here present three possibilities; localized Majorana mass terms, boundary conditions of bulk fields, and curved gravitational backgrounds (and their combinations).

A. Boundary Majorana Masses

We first consider the case that Majorana mass parameters for bulk fermions depend on the extra-dimensional coordinate x^5 . An interesting case is that Majorana masses are localized at the boundaries of fifth dimension ($x^5 = 0, \pi R$). The boundary Majorana masses may be natural if the lepton number symmetry is exact in the bulk and locally broken at the boundaries. For the flat background, the two fixed points are physically equivalent, and in the following we choose $x^5 = 0$ as an example. We consider the Lagrangian

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi - \bar{\Psi}m_d\theta(x^5)\Psi - \left[\frac{1}{2}\bar{\Psi}^c M\Psi + \bar{\Psi}mL + \text{h.c.}\right]\delta(x^5). \quad (4.1)$$

If one also includes the M_5 term, M is replaced with $M + M_5$ in the following formulas. The solutions to the bulk equations of motion have been given in Section II C 2. The mass matrices in four-dimensional effective theory are found

$$M_{K_{mn}} = -\frac{n}{R}\delta_{mn}, \quad M_{R_{mn}} = \frac{f_m f_n}{\pi R}M, \quad M_{L_{mn}} = 0, \quad m_n = \frac{f_n}{\sqrt{\pi R}}m. \quad (4.2)$$

One type of the Majorana masses, M_L , vanishes since N_L^n have the negative Z_2 parity and the wavefunctions become zero at the boundary. Another Majorana mass matrix, M_R , has the off-diagonal ($m \neq n$) entries since the KK momentum is not conserved at the boundary.

It seems difficult to diagonalize the heavy-field mass matrix M_N which is composed of $M_{R,L}$ and the KK masses M_K . However the mixing vertex V between heavy modes and the SM sector can be evaluated by getting the inverse of M_N that is given by

$$V = U_\nu^\dagger \left(\frac{\sqrt{\pi R}}{-f_0} m^\dagger M^{-1} \quad 0 \quad 0 \quad 0 \quad \cdots \right) U_N. \quad (4.3)$$

Notice that all the components but the 1st one are vanishing in the interaction basis. From this mixing matrix, we obtain the seesaw-induced neutrino masses and the heavy-mode mixing with the SM neutrinos:

$$M_\nu = m^\dagger M^{-1*} m, \quad (4.4)$$

$$\nu = U_\nu \nu_d - \frac{\sqrt{\pi R}}{f_0} m^\dagger M^{-1} \epsilon N_R^{0*}. \quad (4.5)$$

The form of M_ν is apparently the same as the usual four-dimensional seesaw mechanism, but ν has an extra factor $\sqrt{\pi R}/f_0$ which originates from the extra dimension, i.e., the

volume factor and the localization factor controlled by the bulk Dirac mass m_d . These two factors are available in the case of boundary Majorana masses and are not obtained for bulk Majorana masses as in the previous section. This is because these factors appear twice in the boundary Majorana masses for bulk fields, but only once in the boundary coupling to the SM fields, which result in the cancellation only for the seesaw-induced masses. In the present case, the two factors can be used for enhancing the heavy-mode couplings to the SM neutrinos, while keeping the seesaw-induced masses un-affected and made tiny. The enhancement by the localization factor requires $-Rm_d \gg 1$. If one introduced Majorana masses at another boundary $x^5 = \pi R$, the bulk mass parameter m_d should be replaced with $-m_d$ in the formula.

In this way, the Majorana masses on the boundary realize an observable seesaw model with appropriate wavefunction factors. The SM fields (neutrinos, electroweak gauge bosons, etc.) interact with the bulk sector only through the zero mode N_R^0 . It is noticed that N_R^0 is not a mass eigenstate, and the cross sections for collider physics might be peaked at the mass eigenvalues of KK neutrinos via the mixing with N_R^0 . This is however not the case in a quantitative meaning: the KK-mode contamination of 1% mixing is found from (4.5) to imply $\sqrt{R}m/(f_0M) \sim \mathcal{O}(10^{-2})$. This in turn implies by the seesaw formula (4.4) that the Majorana mass parameter for N_R^0 is roughly given by $10^4 \times M_\nu$ and very small. Therefore the heavy KK neutrinos do not so much mix with such a light zero mode and cannot be detected at collider experiments.

The conclusion is that, in the extra-dimensional setup in this subsection, the zero-mode wavefunction factors enhance the heavy-mode couplings, keeping the usual seesaw formula, and play a key role for realizing the observable seesaw. However from a phenomenological viewpoint, only the light zero mode is found to be accessible. That depends on which elements are vanishing in the neutrino mass matrices and could be changed by some effects within the model or its extensions. For example, an additional boundary mass or interaction term would lead to a repulsive effect which makes Z_2 -odd fields off from the boundary so that they obtain nonzero Majorana masses. Another option is that a singular boundary profile is regulated by introducing some scalar field, which generates a four-dimensional domain wall with a finite width along the extra dimension. That would lead to non-vanishing couplings of bulk fields in four-dimensional effective theory.

B. Boundary Conditions

Another important option of five-dimensional theory is to choose boundary conditions for bulk fields. For a finite size of extra space, the boundary conditions determine the bulk profile, i.e. wavefunctions of higher-dimensional fields, and then fix their low-energy physics. We have so far discussed the standard boundary condition for a five-dimensional spinor Ψ on the S^1/Z_2 orbifold, that is, the Neumann and Dirichlet type boundary conditions for the

upper and lower components, respectively; $\partial_5 \chi_R^n = \chi_L^n = 0$ at both $x^5 = 0$ and πR . In this section, let us consider another mixed-type condition:⁴

$$\begin{aligned}\Psi(-x^5) &= +\gamma_5 \Psi(x^5), \\ \Psi(-x^5 + 2\pi R) &= -\gamma_5 \Psi(x^5),\end{aligned}\tag{4.6}$$

i.e. the upper component has a positive (negative) parity under the reflection about the $x^5 = 0$ ($x^5 = \pi R$) boundary. The lower component has the opposite parity assignment. In terms of KK-mode wavefunctions, $\partial_5 \chi_R^n(0) = \chi_L^n(0) = 0$ and $\chi_R^n(\pi R) = \partial_5 \chi_L^n(\pi R) = 0$, in the absence of extra boundary terms. Notice that this is equivalent to the Scherk-Schwarz boundary condition [21] where a non-trivial twist is imposed in circulating along the extra dimension: $\Psi(x^5 + 2\pi R) = -\Psi(x^5)$.

Let us consider the following Lagrangian

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi - \left[\frac{1}{2}\bar{\Psi}^c M\Psi + \bar{\Psi}mL\delta(x_5) + \text{h.c.} \right],\tag{4.7}$$

and evaluate the seesaw mass matrix under the boundary conditions (4.6). The wavefunctions for free bulk fields are given by

$$\chi_R^n = \frac{1}{\sqrt{\pi R}} \cos\left[\frac{(n-\frac{1}{2})}{R}x^5\right], \quad \chi_L^n = \frac{1}{\sqrt{\pi R}} \sin\left[\frac{(n-\frac{1}{2})}{R}x^5\right]. \quad (n \geq 1) \tag{4.8}$$

The mass matrices in four-dimensional effective theory are found

$$M_{K_{mn}} = -\frac{n-\frac{1}{2}}{R}\delta_{mn}, \quad M_{R_{mn}} = M_{L_{mn}} = M\delta_{mn}, \quad m_n = \frac{m}{\sqrt{\pi R}}.\tag{4.9}$$

The only difference from the previous standard boundary condition is the KK mass spectrum M_K . We find the seesaw-induced neutrino mass and the heavy-mode mixing with the SM neutrinos:

$$M_\nu = \frac{1}{2\pi R} m^t \frac{\pi R |M|}{\coth(\pi R |M|)} \frac{1}{M^*} m,\tag{4.10}$$

$$\nu = U_\nu \nu_d - \frac{m^\dagger}{\sqrt{\pi R}} \sum_{n=1} \frac{1}{|M|^2 + \left(\frac{n-\frac{1}{2}}{R}\right)^2} \left[\frac{n-\frac{1}{2}}{R} N_L^n + M^* \epsilon N_R^{n*} \right].\tag{4.11}$$

The light neutrino mass M_ν has the factor $\pi R |M| / \coth(\pi R |M|)$ as a consequence of summing up the heavy-mode seesaw contributions. Notice that, for the standard boundary condition, this factor is $\pi R |M| / \tanh(\pi R |M|)$. The difference is understood in the following two limits: For the large radius limit, $RM \gg 1$, the two boundaries are so separated in the

⁴ An overall sign is fixed by assuming that the upper component of Ψ has non-vanishing wavefunction at the boundary $x^5 = 0$ where the SM fields reside.

extra-dimensional space that the difference of boundary conditions at $x^5 = \pi R$ is irrelevant to the SM physics at $x^5 = 0$, and two factors merge into the same value $\pi R|M|$. The other case, $RM \ll 1$, is the decoupling limit of KK modes. They become so heavy that the low-energy physics is determined by light modes only. It is the chiral zero mode in case of the standard boundary condition. For the present twisted boundary condition, the zero mode is absent and the limit $RM \rightarrow 0$ leads to vanishing seesaw-induced masses. That is, the inverse seesaw suppression [22] is realized at each KK level and the total seesaw-induced mass is proportional (not inverse proportional) to heavy-field Majorana mass M .

In this way, the boundary condition mechanism leads to the situation that no massless mode appears in the KK decomposition and therefore bulk Majorana masses can be made small without being conflicting with the heavy-mode integration. Let us consider the case of small Majorana masses ($RM \ll 1$). The seesaw-induced mass and the mass eigenvalues of KK Dirac neutrinos become

$$M_\nu \simeq \frac{\pi R}{2} m^\dagger M m, \quad M_n \simeq \frac{n - \frac{1}{2}}{R} \quad (n \geq 1). \quad (4.12)$$

This agrees with the spectrum of Model 1 discussed in Section II C 1 with the replacement $\delta_M \leftrightarrow M$. The mixing with heavy modes also has the correspondence under this replacement and with a field rearrangement. Therefore the present model with the twisted boundary condition is observable and gives the same seesaw phenomenology, in particular the LHC signatures, as given in Section III. A difference of two models is the interpretation of small parameters δ_M and M . The parameter δ_M in Model 1 is a tiny deviation from the fixed value of model parameter ($M_5 = \frac{1}{2R}$) and is hard to be determined in dynamical way. On the other hand, M is a Lagrangian parameter itself and is easier to be suppressed and controlled with high-energy physics.

If one includes the bulk Dirac mass m_d , the above formulas in low-energy effective theory are modified as

$$M_\nu = \frac{\pi R}{2} m^\dagger \left[-\pi R m_d + \frac{\pi R \sqrt{|M|^2 + m_d^2}}{\tanh(\pi R \sqrt{|M|^2 + m_d^2})} \right]^{-1} M m, \quad (4.13)$$

$$\nu \simeq U_\nu \nu_d - \frac{m^\dagger}{\sqrt{\pi R}} \sum_{n=1} \frac{(n - \frac{1}{2})/R}{[(n - \frac{1}{2})/R]^2 + m_d^2} N_L^n. \quad (4.14)$$

In the regime $-Rm_d \gg 1$, the Dirac mass parameter is effective in suppressing the seesaw-induced masses M_ν , compared with (4.10): for small bulk Majorana masses, we obtain $M_\nu \simeq m^\dagger (M/4m_d) m$.

C. Boundary Majorana Masses and Boundary Conditions

An interesting and physically different scheme is given by considering both of boundary Majorana mass and non-trivial boundary condition of bulk neutrinos, discussed in the

previous two sections. This model is particular in that the seesaw-induced neutrino mass vanishes for any values of model parameters. Therefore the heavy-mode couplings to the SM sector are arbitrarily fixed so that the scenario is observable at collider experiments. The Majorana mass parameters do not appear in any place of low-energy effective theory at the leading order.

Let us consider the same Lagrangian as in Section IV A

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi - \left[\frac{1}{2}\bar{\Psi}^c M \Psi + \bar{\Psi} m L + \text{h.c.} \right] \delta(x^5). \quad (4.15)$$

That is, the Majorana masses for bulk fermions are only on the SM boundary. Further we assume the twisted boundary condition as in Section IV B:

$$\begin{aligned} \Psi(-x^5) &= +\gamma_5 \Psi(x^5), \\ \Psi(-x^5 + 2\pi R) &= -\gamma_5 \Psi(x^5). \end{aligned} \quad (4.16)$$

Therefore the wavefunctions and KK masses are given by (4.8) as previously. The mass matrices in four-dimensional effective theory are found

$$M_{K_{mn}} = -\frac{n - \frac{1}{2}}{R} \delta_{mn}, \quad M_{R_{mn}} = \frac{1}{\pi R} M, \quad M_{L_{mn}} = 0, \quad m_n = \frac{m}{\sqrt{\pi R}}. \quad (4.17)$$

The Majorana masses M_L vanish since N_L^n have the negative Z_2 parity and the wavefunctions become zero at the $x^5 = 0$ boundary on which the Lagrangian mass term is placed. Another Majorana mass matrix M_R takes the common value for all the matrix elements including the off-diagonal ones. The vertex matrix V of heavy modes can be evaluated by taking the inverse of M_N that is given by

$$V = -\sqrt{\frac{4R}{\pi}} U_\nu^\dagger m^T \left(0 \quad 1 \quad 0 \quad \frac{1}{3} \quad 0 \quad \frac{1}{5} \quad 0 \quad \cdots \right) U_N. \quad (4.18)$$

Notice that the $(2n - 1)$ -th components are all vanishing in the interaction basis of KK modes. Further the non-vanishing elements do not depend on the Majorana mass parameter M . From this mixing matrix, we find the seesaw-induced neutrino mass and the heavy-mode mixing with the SM neutrinos:

$$M_\nu = 0, \quad (4.19)$$

$$\nu = U_\nu \nu_d - \sqrt{\frac{R}{\pi}} m^\dagger \sum_{n=1} \frac{2}{2n-1} N_L^n. \quad (4.20)$$

It is interesting that the light neutrino mass M_ν vanishes, irrespectively of model parameters. The heavy-mode mixing is governed by the compactification scale and the boundary mass m . Their ratio can therefore be arbitrarily fixed and made sizable. In this model, the bulk Majorana mass M does not join in any formula of the seesaw operation and only affects the

mass spectrum of heavy modes. The spectrum is found to be roughly determined only by the compactification scale and may be corrected by Majorana masses which are suppressed by the cutoff scale of the theory.

The above result shows that the scheme in this subsection gives a natural realization of the observable seesaw in the zero-th approximation. Towards a phenomenologically viable model, nonzero neutrino masses are needed to be generated by some dynamics. Among various possibilities, a simple way is to put, as a correction, the Majorana masses in the bulk and/or on the other boundary $x^5 = \pi R$:

$$\Delta\mathcal{L} = -\frac{1}{2}(\overline{\Psi^c}M_b\Psi + \text{h.c.}) - \frac{1}{2}(\overline{\Psi^c}M_\pi\Psi + \text{h.c.})\delta(x^5 - \pi R). \quad (4.21)$$

Repeating the previous procedure with these terms, we obtain the seesaw-induced neutrino masses

$$\Delta M_\nu = \frac{1}{2\pi R} m^\dagger (\pi R)^2 \left(M_b + \frac{1}{\pi R} M_\pi \right) m + \mathcal{O}(RM_b, RM_\pi). \quad (4.22)$$

Finally we briefly comment on other patterns of the model. There seems to exist 3 degrees of freedom: the boundary Majorana masses on $x^5 = 0$ or πR , the SM fields reside at $x^5 = 0$ or πR , and the choice of boundary condition (the overall sign of Z_2 parity assignment). However an actual freedom is only one, because two boundaries are equivalent in the flat background, and the upper and lower components of bulk fermions can be appropriately exchanged. As the remaining freedom, let us consider the situation that boundary Majorana masses are placed at $x^5 = \pi R$, instead of $x^5 = 0$ discussed before. The boundary condition is the same as previously and we then find

$$M_\nu = \frac{1}{2\pi R} m^\dagger \left(\frac{\pi R M}{2} \right) m, \quad (4.23)$$

$$\nu = U_\nu \nu_d - \sqrt{\frac{R}{\pi}} m^\dagger \sum_{n=1} \left[\frac{2}{2n-1} N_L^n - \frac{(-1)^n}{2n-1} M^* \epsilon N_R^{n*} \right]. \quad (4.24)$$

The contribution of N_L^n does not depend on the Majorana mass parameter M , and therefore the observable seesaw is realized for a suitably value of M for obtaining tiny seesaw-induced masses, while keeping the N_L^n mixing sizable. The heavy neutrinos are degenerate in mass and their spectrum is almost given by the KK masses $(n - \frac{1}{2})/R$ ($n \geq 1$).

D. AdS₅ Gravitational Background

So far we have discussed the bulk Lagrangian on the flat gravitational background. Another typical geometry of extra dimension is given by the so-called AdS₅ warped background [23]. That is a solution of the Einstein equation in the five-dimensional theory with

appropriately tuned cosmological constants both in the bulk and on the boundaries. The line element is

$$ds^2 = e^{-2k|x^5|} \eta_{\mu\nu} dx^\mu dx^\nu - dx_5^2, \quad (4.25)$$

where k is the AdS curvature and is related to the bulk cosmological constant. Neutrino physics on the warped geometry has been studied in the absence of Majorana masses [24]. Let us consider the bulk field Lagrangian on this background. Evaluating the spin connection and normalizing kinetic term ($\Psi \rightarrow e^{\frac{3}{2}k|x^5|}\Psi$), we have

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi - e^{-k|x^5|} \left[\bar{\Psi}\gamma_5\partial_5\Psi + \bar{\Psi}\left(m_d - \frac{k}{2}\gamma_5\right)\theta(x^5)\Psi + \frac{1}{2}(\bar{\Psi}^c M\Psi + \text{h.c.}) \right]. \quad (4.26)$$

The bulk Dirac and Majorana mass terms depend on the extra-dimensional coordinate x^5 which arise from the warped metric. In the absence of bulk mass terms, the solutions to the bulk equations of motion are given by

$$\chi_R^n = \frac{h_n}{\sqrt{\pi R}} e^{\frac{1}{2}k|x^5|} \cos\left[\frac{nh_n^2}{kR}(e^{k|x^5|} - 1)\right], \quad (4.27)$$

$$\chi_L^n = \frac{h_n}{\sqrt{\pi R}} e^{\frac{1}{2}k|x^5|} \sin\left[\frac{nh_n^2}{kR}(e^{k|x^5|} - 1)\right], \quad (4.28)$$

where the normalization factor is given by $h_n = \sqrt{\pi k R / 2^{\delta_{n0}}(e^{\pi k R} - 1)}$ and the massive mode spectrum is $n\pi k / (e^{\pi k R} - 1)$ ($n = 1, 2, \dots$).

We assume that the SM fields live on the infrared ($x^5 = \pi R$) boundary where the fundamental scale is reduced to TeV and the hierarchy problem is solved if $kR \sim 10$ [23]. It is a non-trivial task to obtain the analytic expression of seesaw-induced masses by evaluating the mass matrix elements and summing up the contributions of KK-mode integration. Here we consider a simple and tractable case that Majorana masses are given only on the $x_5 = 0$ boundary [25] (and bulk Dirac masses vanish, just for simplicity), though the result given below does not depend on whether the Majorana masses are placed on the $x^5 = 0$ or $x^5 = \pi R$ boundary. Using the above wavefunctions, we obtain the mass matrices in four-dimensional effective theory;

$$M_{Kmn} = -\frac{n}{R} h_n^2 \delta_{mn}, \quad M_{Rmn} = \frac{h_m h_n}{\pi R} M, \quad M_{Lmn} = 0, \quad m_n = \frac{e^{\frac{1}{2}\pi k R} h_n}{\sqrt{\pi R}} m. \quad (4.29)$$

One type of the Majorana masses, M_L , vanishes since N_L^n have the negative Z_2 parity and the wavefunctions become zero at the boundary. Another Majorana mass matrix, M_R , has the off-diagonal ($m \neq n$) entries since the KK momentum is not conserved at the boundary. The KK and Majorana masses receive the exponential warp factors from the gravitational background.

It seems difficult to diagonalize the heavy-field mass matrix M_N which is composed of M_R and the KK masses M_K . However the mixing vertex V between heavy modes and the SM fields can be evaluated by taking the inverse of M_N that is given by

$$V = U_\nu^\dagger \left(\frac{\sqrt{\pi R}}{-h_0} e^{\frac{1}{2}\pi k R} m^\dagger M^{-1} \quad 0 \quad 0 \quad 0 \quad \cdots \right) U_N. \quad (4.30)$$

Notice that all the components but the 1st one are vanishing in the interaction basis of heavy modes. From this mixing matrix, we obtain the seesaw-induced masses and the heavy-mode mixing with the SM neutrinos:

$$M_\nu = e^{\pi k R} m^\dagger M^{-1*} m, \quad (4.31)$$

$$\nu \simeq U_\nu \nu_d - \sqrt{\frac{2}{k}} e^{\pi k R} m^\dagger M^{-1} \epsilon N_R^{0*}. \quad (4.32)$$

It is found that the result is almost the same as in the standard four-dimensional seesaw model with the heavy mass scale $M' = M e^{-\pi k R} \sim \text{TeV}$. Therefore the model on the warped extra dimension cannot naively be made observable at collider experiments because the heavy-mode mixing to the SM sector is roughly given by $|V| \sim (M_\nu/M')^{1/2} \sim 10^{-6}$ and is too small to be detected. The conclusion would not be changed even if different curved geometries are considered because the seesaw-induced mass is determined without the knowledge of background metric [26].

An introduction of bulk Dirac masses modifies the wavefunctions of bulk fermions Ψ . It is easily found that the Dirac masses lead to additional wavefunction factors, which is similar to the result in Section IV A, and hence does not cure the problem. Another option is to extend the SM fields into the five-dimensional bulk and to include their bulk Dirac masses. In low-energy effective theory, the mixing elements m_n between the SM neutrinos and bulk singlets are suppressed if one chooses the bulk masses of left-handed leptons such that they are localized away from the boundary at which the neutrino Yukawa coupling is given. However even in this case, we could not have the observable seesaw model in the sense that only the zero mode is accessible and the higher KK-mode mixing with the SM neutrinos is small.

Finally we comment on a possible modification of Model 2 given in Section II C 2 by considering the same field configuration on the warped background. Unlike the flat background, the mass eigenvalues of low-lying KK-excited modes are not dominated by the bulk Dirac mass m_d and their wavefunctions become un-suppressed. In addition, the KK-excited modes are localized towards the $x_5 = \pi R$ boundary and have stronger couplings to the SM sector. These facts would make it possible to observe the right-handed neutrinos at the LHC.

V. SUMMARY

We have presented several seesaw scenarios in a five-dimensional extension of the SM, where right-handed neutrinos live in the bulk and the SM particles stay at a four-dimensional

boundary. The light neutrino mass scale is of the order of eV, while the TeV-scale KK neutrino modes have sizable gauge and Yukawa couplings to the SM sector, which situation leads to observable signatures in future particle experiments. We have discussed various extra-dimensional schemes for making heavy states in the seesaw mechanism observable. Among them, the collider signatures have been analyzed for two illustrative models: the one involves the seesaw cancellation with a particular value of bulk Majorana mass and another has light Dirac neutrinos. Both models realize approximate lepton number conservation in low-energy effective theory.

As the most effective LHC signal, we have analyzed the processes with tri-lepton final states $pp \rightarrow \ell^\pm \ell^\pm \ell^\mp \nu(\bar{\nu})$ and its conjugates. We have extended our previous study to including three types of neutrino mass patterns allowed by the current experimental data. It is found that the scenarios give excessive tri-lepton events beyond the SM background in wide regions of parameter space and the LHC would discover a sign of tiny neutrino mass generation. Further, as for the three-generation mixing, the cross sections are controlled by the MNS neutrino mixing matrix, and therefore a detailed measurement of branching ratios would corroborate the lepton flavor structure and the Type I seesaw scheme, which is left for future study.

In the present analysis, the signal essentially receives the contribution only from the 1st excited mode and is difficult to discriminate the seesaw mechanism in higher dimensions from other models for neutrino mass generation. The observation of higher KK modes is expected to be within the reach of future particle experiments such as the ILC. That could substantially confirm the existence of extra spatial dimensions in Nature.

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